# The Nucleon Axial-Vector Form Factor for Precision Neutrino Oscillation Studies

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June 14, 2016

#### Motivation

$$\Phi(E_{\nu}) = \frac{\mathcal{N}(E_{\nu})}{\sigma_{A}(E_{\nu})}$$

Oscillation experiments monitor flux by counting interactions assuming cross section, near/far detector do not perfectly cancel

⇒ Measurements of neutrino oscillation depend on precise knowledge of neutrino cross section

$$\sigma_A \sim \sigma_{CCQE} \otimes (\text{nucl. models})$$

 $(\sigma_{CCQE}(E_{\nu},Q^2))$  is quadratic function of form factors)

- Nuclear effects entangled with nucleon amplitudes
   factorization is oversimplification
- Model-dependent shape parameterization introduces systematic uncertainties and underestimates errors

# Discrepancies in the Axial-Vector Form Factor

 $\sigma_{CCQE}$  dependent on form factors:  $F_{1V}(Q^2)$ ,  $F_{2V}(Q^2)$ ,  $F_{A}(Q^2)$ ,  $F_{P}(Q^2)$ 

Most analyses assume the "Dipole form factor":

$$F_A^{ ext{dipole}}(Q^2) = g_A rac{1}{\left(1 + rac{Q^2}{m_A^2}
ight)^2}$$

Dipole is an ansatz:

unmotivated in interesting  $Q^2$  (4-momentum) region ⇒ uncontrolled systematics and underestimated uncertainties

Large variation in  $m_A$  over many experiments:  $m_{\Delta}^{\rm eff} = 1.35 \pm 0.17$  (MiniBooNE, 1002.2680[hep-ph])  $m_A = 1.026 \pm 0.021$  world avg. QE (Bernard et. al, 0107088[hep-ph])

Essential to replace with model-independent parameterization

### z-Expansion

The z-Expansion (Bhattacharya, Hill, Paz arXiv:1108.0423 [hep-ph]) is a conformal mapping which takes the kinematically allowed region ( $t \le 0$ ) to within  $z = \pm 1$ 

$$z(t; t_0, t_c) = \frac{\sqrt{t_c - t} - \sqrt{t_c - t_0}}{\sqrt{t_c - t} + \sqrt{t_c - t_0}} \qquad F_A(z) = \sum_{n=0}^{\infty} a_n z^n$$

$$(t = q^2 = -Q^2, t_c = 9m_{\pi}^2)$$

$$\frac{|t|}{-Q_{\max}^2} \frac{|z|}{9m_{\pi}^2}$$

# Advantages of z-Expansion

- z-Expansion is a model-independent description of the axial form factor
  - Motivated by analyticity arguments
  - Only a few coefficients needed to accurately represent form factor
  - Provides a prescription for introducing more parameters as data improves
  - Allows quantification of systematic errors
  - Coefficient falloff required by perturbative QCD

# Deuterium Fitting (1603.03048[hep-ph])

with Richard Hill, Rik Gran, Minerba Betancourt

Fits to deuterium bubble chamber data (relatively small nuclear effects)

#### Three datasets:

- ANL 1982: 1737 events, 0.5GeV [peak]
- BNL 1981: 1138 events, 1.6 GeV [average]
- FNAL 1983: 362 events, 20 GeV [peak], 27 GeV [average]

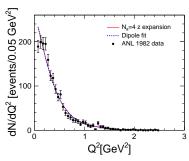
Shape-only fits to QE differential cross section data

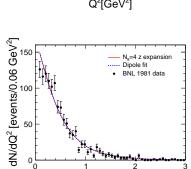
Gaussian priors used on z-Expansion coefficients:

if (k 
$$\leq$$
 5)  $\sigma_k =$  5, else  $\sigma_k = 25/k$ 

Sum rules applied to enforce large  $Q^2$  falloff

## Deuterium Fits - Differential Cross Section





Q<sup>2</sup>[GeV<sup>2</sup>]

#### Dinolo:

Dipole.	
$\chi^2/N_{bins}$	58.6/49
$\overline{m_A}$	1.02(5)

#### z-Expansion:

= = 1, p a				
$\chi^2/N_{\rm bins}$	60.9/49			
$a_1$	2.25(10)			
$a_2$	0.2(0.9)			
$a_3$	-4.9(2.4)			
a₄	2.7(2.7)			

# Dipole:

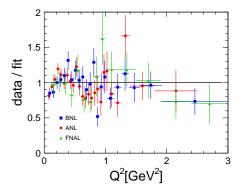
$\dot{\chi}^2/N_{\rm bins}$	70.9/49
$m_A$	1.05(4)

#### z-Expansion:

$\chi^2/N_{\rm bins}$	73.4/49		
$a_1$	2.24(10)		
$a_2$	0.6(1.0)		
$a_3$	-5.4(2.4)		
⁴ □ å₄ ⁴ 🗗 🕨	2.2(2.7)		

#### Residuals

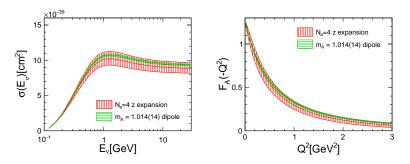
Residuals indicate potentially correlated effect between experiments



Neither z expansion, nor dipole can properly explain shape of data  $\implies$  underestimated systematic effects

#### Final Fits

Final fits include systematics of acceptance corrections, deuterium nuclear corrections



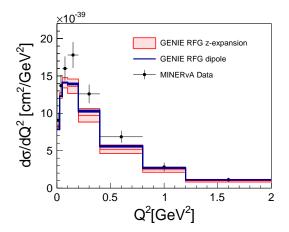
Calculated observables:

$$r_A^2 = 0.46(22) \, \mathrm{fm}^2 \,, \quad \sigma_{\nu n \to \mu p}(E_{\nu} = 1 \, \mathrm{GeV}) = 10.1(0.9) \times 10^{-39} \mathrm{cm}^2$$
 compared with Bodek *et. al* [Eur. Phys. J. C 53, 349]:

$$r_A^2 = 0.453(13) \, \text{fm}^2 \,, \quad \sigma_{\nu n \to \mu p}(E_{\nu} = 1 \, \text{GeV}) = 10.63(0.14) \times 10^{-39} \text{cm}^2$$

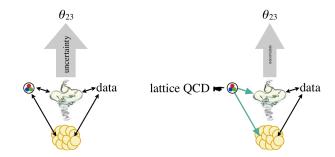
### z Expansion in GENIE

To be officially released in production version 2.12 Currently available in GENIE "trunk" version



# Lattice QCD in Neutrino Physics

- LQCD measurements becoming more accurate, precise
   ⇒ now able to inform neutrino experiment
- LQCD enables clean measurement of form factors (no nuclear corrections, no experiment systematics)
- Offers way of breaking measurement degeneracy between nuclear models, nucleon form factors
- Less explosive than hydrogen!

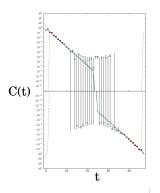


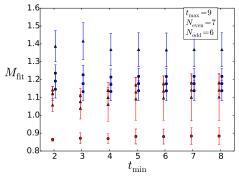
#### Current Lattice Effort

LQCD calculation of form factors underway by MILC/Fermilab Lattice Collaborations

Lattice computation involves several stages, building up to result: 2-point functions = masses, overlap factors

$$\lim_{t \to \infty} \langle N(0) | N(t) \rangle \sim |a|^2 e^{-m_N t}$$

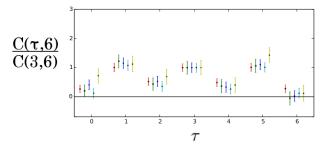




## Lattice QCD Axial Form Factor

Use 2-point functions to calculate 3-point functions = form factors

$$\lim_{\tau,t\to\infty} \left\langle N'(0) |\, A_\mu(x,\tau) \, |\, N(t) \right\rangle \sim F_A(Q^2) |a|^2 e^{-m_N \tau} e^{-m_N (t-\tau)} e^{-iq\cdot x}$$



Ratio taken  $\rightarrow$  poor-man's blinding

#### Conclusions

#### Neutrino physics is subject to underestimated and model-dependent systematics

- ightarrow To reduce systematics from modeling, need to understand nuclear physics
- → To understand nuclear physics, need to understand nucleon-level cross sections from an ab initio calculation
  - z-Expansion removes model assumptions and permits better understanding of systematic errors
  - hydrogen (deuterium) targets have relatively small nuclear effects
  - LQCD offers a way to access nucleon form factors directly

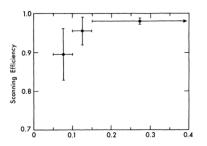
# Thanks!

# Backup Slide(s)

# **Acceptance Corrections**

Acceptance correction for fixing errors from hand scanning  $Q^2$  dependent correction, correlated between bins:

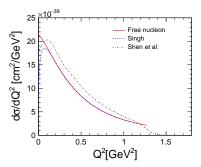
$$rac{dN}{e(Q^2)} 
ightarrow rac{dN}{e(Q^2) + \eta \, de(Q^2)} \, , \quad \eta = 0 \pm 1$$



For ANL, BNL, FNAL respectively,  $\eta = -1.9, -1.0, +0.01$ ; minimal improvement of goodness of fit

#### **Deuterium Corrections**

Corrections assumed to be  $E_{\nu}$  independent Two corrections tested: Singh Nucl. Phys. B 36, 419, Shen 1205.4337 [nucl-th]



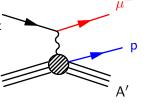
Central values of Shen, Singh are consistent with each other Final fit done with Singh, inflated error bars

#### **Nuclear Effects**

Nuclear effects not well understood

→ Models which are best for one measurement are worst for another

Need to break  $F_A$ /nuclear model entanglement



(assumed  $m_A = 0.99$  GeV)

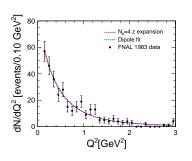
 $\nu_{\mu}$ 

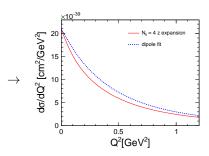
NuWro Model	RFG	RFG+	assorted
$(\chi^2/DOF)$	[GENIE]	TEM	others
leptonic(rate)	3.5	2.4	2.8-3.7
leptonic(shape)	4.1	1.7	2.1-3.8
hadronic(rate)	1.7[1.2]	3.9	1.9-3.7
hadronic(shape)	3.3[1.8]	5.8	3.6-4.8

(Minerva collaboration, 1305.2243,1409.4497[hep-ph])

# Normalization Degeneracy

Despite similarity of dipole/z expansion, cross sections not the same





Consequence of self-consistency: cross section prediction

$$\frac{dN}{dE} \propto \frac{1}{\sigma} \frac{d\sigma}{dQ^2}$$

Cross section shape controlled by low- $Q^2$  data, normalization